A Convexity Proof for the Partition Function of the Trace Norm Distribution*

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Abstract

The partition function for the Trace Norm Distribution involves a product of polynomial terms of the singular values. We show that this function is unimodal over the feasible region.

1 Introduction

An important quantity in the trace norm distribution partition function is [1],

$$J \equiv \int_0^\infty \int_0^{\sigma_2} \cdots \int_0^{\sigma_{m-1}} \prod_{i=1}^m e^{-\lambda\sigma_i} \sigma_i^{n-m} \prod_{i<j} (\sigma_i^2 - \sigma_j^2) d\sigma_m \ldots d\sigma_2 d\sigma_1. \quad (1)$$

Consider the distribution over singular values defined by the inner quantity,

$$P(\sigma_1 \geq \cdots \geq \sigma_m \geq 0) \propto \prod_{i=1}^m e^{-\lambda\sigma_i} \sigma_i^{n-m} \prod_{i<j} (\sigma_i^2 - \sigma_j^2). \quad (2)$$

This distribution has a single mode as a function of the singular values. The normalizer is not a function of the singular values, and the log-probability is a sum of concave functions,

$$\log P = \sum_{i=1}^m [(n-m)\log\sigma_i - \lambda\sigma_i] + \sum_{i<j} [\log(\sigma_i + \sigma_j) + \log(\sigma_i - \sigma_j)] - \log Z_\lambda. \quad (3)$$

Hence, $\log P$ is concave over $\sigma_1 \geq \cdots \geq \sigma_m \geq 0$. Thus, P has a single mode, which is its unique maximum.

One way to approximate J is to approximate P (such as use a simpler distribution with the same initial modes), then use importance sampling to estimate J.

*Joint work with John Barnett and Tommi Jaakkola.

References

[1] J. D. M. Rennie. Computing the trace norm distribution via sampling. http://people.csail.mit.edu/~jrennie/writing, January 2006.